Indian Statistical Institute

B. Math. Hons. III Year

Semestral Examination 2008-2009

Topology

Duration: 3 Hours | Marks: 50 | Instructor: J. Biswas

Attempt all questions, each carries 10 marks.

1. State whether each of the following statements is true or false and give your reasons for the answer. You may quote any theorem which has been taught in class.

   a)The fundamental group of $\mathbb{R}^3$ with nonnegative x, y and z axes deleted is abelian.

   b)Every continuous map $f : S^1 \to P^2$ is nullhomotopic.

   c)A covering map is a closed map.

2. Let $p : E \to B$ be a covering map such that $p^{-1}(b)$ is a finite set for all $b \in B$. Prove that B is compact, Hausdorff if and only if E is compact, Hausdorff.

3. a) Let $p : E \to B$ be a covering map where E is the figure eight. What can you say about B?

   b) Let $X \subset \mathbb{R}^3$ be the union of S^2 and a diameter of S^2. Construct a simply connected covering space of X.

4. Prove that S^n is a retract of the closed unit disc B^{n+1}, if and only if, S^n is contractible.

5. Let $X \subset \mathbb{R}^2$ be the union of the four sides of the square $[0,1] \times [0,1]$ together with the line segments $\{1/n\} \times [0,1]$ for all $n \in N$. Show that, for every covering space $E \to X$, there is some neighbourhood of the left edge $\{0\} \times [0,1]$ in X which lifts homeomorphically to E. Deduce that X has no simply connected covering space.